Introduction to Geostatistics

10. Correlation and regression

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summer semester 2007/8,
June 12, 2008

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Correlation and regression

t-tests and analysis of variance look at how a single continuous variable depends on a categorical variable with two levels (t-test), more levels (one-way anova), or on more categorical variables (more-way anova).
The focus now shifts to the relation between two (or more) continuous variables. We start with how one continuous variable depends on another dependent variable.

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sample and population correlation

We can compute sample correlation,

```
> cor(Length, Weight)

[1] 0.6797413
```

but also test whether the population correlation (ρ) has a certain value. Typically, $H_0 : \rho = 0$.

```
> cor.test(Length, Weight)

        Pearson's product-moment correlation

data:  Length and Weight
t = 8.494, df = 84, p-value = 6.19e-13
alternative hypothesis: true correlation is not equal to 0
95 percent confidence interval:
 0.5465855 0.7793713
sample estimates:
      cor
0.6797413
```

correlation: symmetry

As can be glanced from the equation how to compute correlation,

$$r(X, Y) = \frac{\text{Cov}(X, Y)}{\sqrt{\text{Var}(X)\text{Var}(Y)}}$$

it is true that $r(x, y) = r(y, x)$. Indeed,

```
> cor(Length, Weight)

[1] 0.6797413

> cor(Weight, Length)

[1] 0.6797413
```

Linear regression

Regression looks at asymmetric problems, where one variable depends on another. E.g. in simple linear regression, for n observations y_i, $i = 1, ..., n$:

$$y_i = \beta_0 + \beta_1 x_i + e_i$$

with e a zero-mean random variable, β_0 and β_1 unknown but non-random population parameters, and X known. So,

$$\mathrm{E}(y_i) = \beta_0 + \beta_1 x_i$$

As e is random, it means that y is random as well, whereas x is not.

A test the regression slope

The typical problem in looking at linear relationships between two continuous variables, is to ask oneself whether one variable depends on the other. Dependence is a rather broad concept, and can have many forms. We usually first look at whether one variable linearly depends on the other, as in

$$y_i = \beta_0 + \beta_1 x_i + e_i$$

If this dependence is not the case, then $\beta_1 = 0$. So, this is the typical H_0 for this kind of test.

How to estimate the parameters?

Under the assumptions that

(i) the observations are independent (and consequently the e_i are independent) and

(ii) that the variance of e_i is constant,

the best estimates for β_0 and β_1 are obtained by minimizing the sum of squared regression residuals, $\sum_{i=1}^{n} e_i^2$: and are

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$

Let's do it (with R) – 1

```
> lm(Weight ~ Length)

Call:
lm(formula = Weight ~ Length)

Coefficients:
(Intercept)       Length
   -145.998        1.222
```

The intercept refers to the value of y when x is zero, the value called `Length` to the regression coefficient that belongs to variable `Length`. Thus, the equation for the regression line is:

$$\mathrm{E}(\texttt{Weight}) = -145.998 + 1.222\ \texttt{Length}$$

>

Let's do it (with R) – 2

```
> summary(lm(Weight ~ Length))

Call:
lm(formula = Weight ~ Length)

Residuals:
    Min      1Q  Median      3Q     Max
-18.555  -9.143  -2.674   5.003  77.239

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) -145.9984    25.6926  -5.683 1.87e-07 ***
Length         1.2216     0.1438   8.494 6.19e-13 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 15.38 on 84 degrees of freedom
Multiple R-squared: 0.462,        Adjusted R-squared: 0.4556
F-statistic: 72.15 on 1 and 84 DF,  p-value: 6.19e-13
```

A model for the data

For each data point y_i, we can decompose the difference from the mean of y, $\bar{y}$ as

$$y_i - \bar{y} = (y_i - \hat{y}) + (\hat{y} - \bar{y})$$

As the two right-hand side terms are independent, we can write this as

$$(y_i - \bar{y})^2 = (y_i - \hat{y})^2 + (\hat{y} - \bar{y})^2$$

and summed over all measurements:

$$SS_{tot} = SS_{resid} + SS_{reg}$$

```
> summary(aov(Weight ~ Length))

            Df  Sum Sq Mean Sq F value   Pr(>F)
Length       1 17071.2 17071.2  72.148 6.19e-13 ***
Residuals   84 19875.6   236.6
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

- `Residual standard error: 15.38`: this is the square-root of `MS Residuals` (236.6)
- `on 84 degrees of freedom`: $n - 2$ (two coefficients were estimated: β_0 and β_1, to obtain residuals)
- `Multiple R-squared: 0.462` this is SS_{reg}/SS_{tot}, a measure between 0 and 1, where 1 indicates a perfect fit, 0 absence of fit
- `Adjusted R-squared: 0.4556` forget for now
- `F-statistic: 72.15 on 1 and 84 DF` the ration of the mean squares (MS_{reg}/MS_{resid})
- `p-value: 6.19e-1` the p-value of the test for the slope, on $H_0 : \beta_1 = 0$

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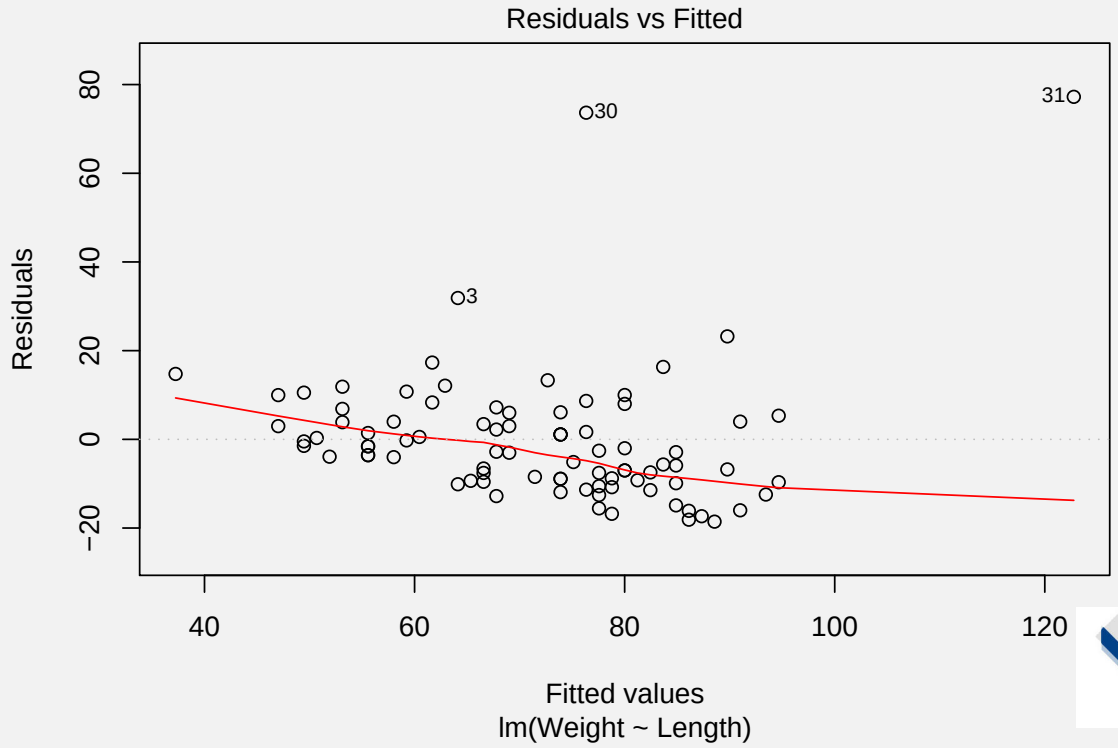
Diagnostic plots, 1

```
> plot(lm(Weight ~ Length), which = 1)
```



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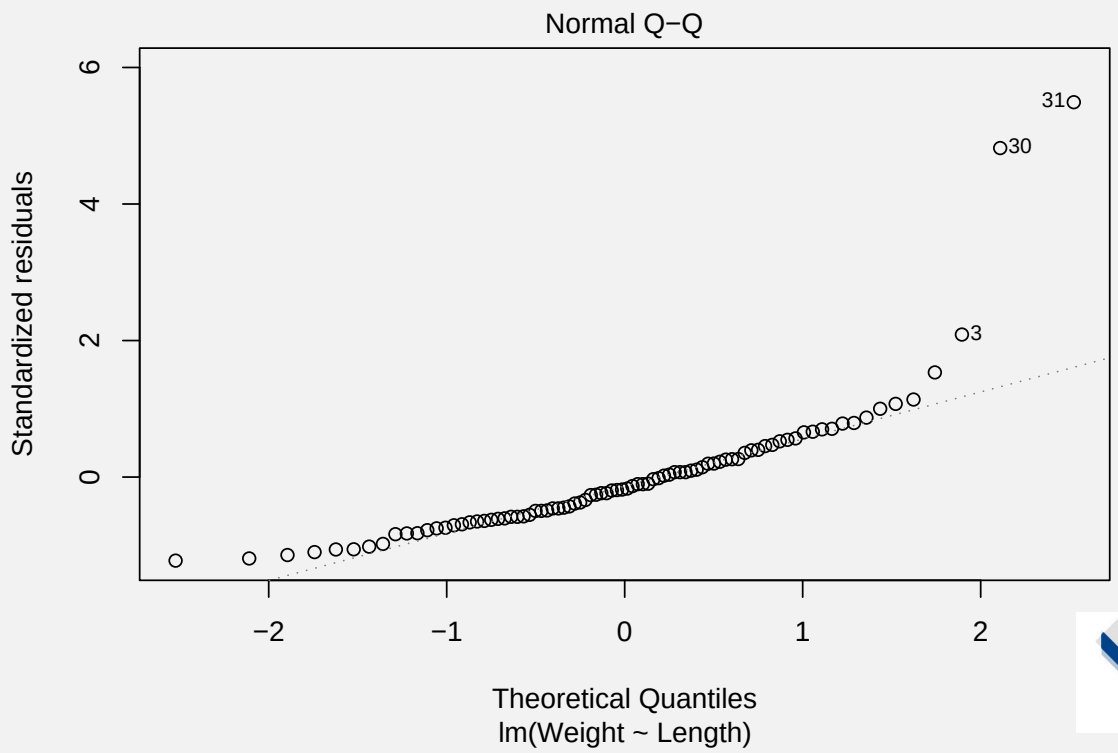
Diagnostic plots, 2

```
> plot(lm(Weight ~ Length), which = 2)
```



Diagnostic plots, 3

```
> plot(lm(Weight ~ Length), which = 3)
```